Structural Analysis of Conveyor Belts. II. Finite Element Approach

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SYNOPSIS

The use of finite element analysis (FEA) in the design of a conveyor belt reduces the number of assumptions required in other methods. Although the problem is nonlinear (because of material property and geometry), a linear analysis produces comparable results. For handling the problem of large displacements, a prestrain method based upon a temperature gradient was used. The thickness of rubber interplies corresponding to minimum shear stress were determined for a number of three-ply belts. © 1992 John Wiley & Sons, Inc.

INTRODUCTION

The purpose of structural analysis of a conveyor belt is to determine the state of all stress components acting throughout the belt. The end result of such analysis is the calculation of a load vector that after modification with an appropriate safety factor could be used to determine the required properties of the belt materials. In this analysis, the calculated stress field should represent a system of external and internal forces in equilibrium throughout the body with continuous displacements (the condition of compatibility).

To determine the state of stresses and displacements, the governing equations must be clearly defined. Apart from the problem of solving the chosen equations, the main difficulty lies in the ability of the equations to represent truly the design conditions. Complications in geometry, loading, and material properties should also be taken into consideration. For such equations, an exact solution rarely exists and many assumptions and approximations are required for an accurate solution. The following are usually assumed[1]:

1. The material properties are linear (although fabric may have different moduli at tension and compression.
2. Changes in geometry do not affect the solution.
3. Rubber does not carry longitudinal stress.
4. Shear deformation of the fabric is negligible.

The above assumptions can be reduced to the first two using linear finite element analysis (FEA) and may totally be eliminated by a nonlinear method.

Most commercially available FEA programs are based upon the stiffness matrix method, i.e.

$$[\mathbf{K}]\{\mathbf{U}\} = \{\mathbf{F}\} \tag{1}$$

where $[\mathbf{K}]$ is the total stiffness matrix, $\{\mathbf{U}\}$ the displacement vector, and $\{\mathbf{F}\}$ the external applied force vector. With the rubber stiffness being rate dependent, nonlinear effects are dominant; thus

$$[\mathbf{K}(\sigma, \mathbf{U})]\{\mathbf{U}\} = \{\mathbf{F}\} \tag{2}$$

An ordinary linear FEA is unable to evaluate the problems concerning incompressibility of rubber unless a strain energy potential function is used instead of stiffness matrix.[2] Besides, modeling of friction elements is not in the capability of linear FEA programs.[3]

Despite its limitations, there are applications where the linear FEA is the correct choice. It provides useful information about deflection and also good information about location, direction, and relative magnitude of stresses.[3] A linear FEA is considered to be a valid choice when a parametric study

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is to be performed, especially in determining the effects of various changes.[4]

The bimodular property of the fabric makes it almost impossible to be evaluated completely with either linear or nonlinear analysis. Fabric is made up of fibers that are assumed to be transversely isotropic, but this may not be true for the fabric itself. The yarns of fabric in warp and weft may be of different materials. This causes the fabric to be orthotropic. If we assume the fibers to be orthotropic rather than isotropic, a three-dimensional analysis is required for the material properties in the three directions. Such analysis, however, is rather time consuming.

FEA of rubber products such as seals, bearings, etc. have been widely published,[5] often with experimental proof. Most work on tires are also performed using an FEA approach, but no such analysis of conveyor belts has yet been reported.

The purpose of this article is to evaluate the proper rubber interply thickness of a conveyor belt. A linear FEA is used to estimate the effect of this parameter upon the resultant shear stress on the rubber and longitudinal stresses on the fabric.

DESCRIPTION OF THE MODEL

Conveyor belts are usually made of a few number of fabric (or steel) plies joined together by rubber interplies with top and bottom rubber covers. It has been found that in a conveyor belt maximum stresses appear when the belt bends around the drive pulley,[1] a point that should be taken into considerations in design. The main complications to be considered in an FEA approach are:

1. The belt is a composite of fabric and rubber.
2. Rubber and fabric materials are nonlinear.
3. Rubber is nearly incompressible ($\nu \approx 0.5$).
4. Friction elements must be modeled in this analysis.
5. Bending of the belt around the pulley causes large displacement.
6. A three-dimensional analysis is time consuming.

Considerable simplification of the above points can be achieved through proper arguments.

First, if the amount of rubber "frictioned" into the fabric is considered low (which is not unreasonable for the case of finely woven fabrics such as those of EP100-200) then each individual layer of fabric and rubber can be regarded as separate.

Second, the actual load acting on the carcass of the conveyor belt is approximately $\frac{1}{18}$ of the design load. In this range, the applied stresses are low enough so that the strains may not exceed 3%. At such low strains, the nonlinear behavior of the belt constituents is negligible and hence the fabric and rubber bonded to it can be considered linear in tension.

Third, the linear elastic constitutive relationship is incapable of accounting for the incompressibility of the rubber due to the well-known singularity in the material stiffness matrix. Attempts to approximate the Poisson ratio of the rubber to slightly lower values (e.g., 0.499 or 0.49) results in numerical difficulties.[2] For instance, taking a value of 0.49 for the Poisson ratio predicts the stresses in error by a factor of almost two.[3] Such inaccuracy is, however, acceptable in a parametric study such as the present work, in which the results are judged relatively to one another. Furthermore, since the magnitude of forces and displacements throughout the thickness and across the width of the belt are not of major importance the Poisson ratio can be selected in a manner such that the total stiffness matrix not be ill conditioned.

Fourth, the pulley itself is not under consideration and only its effects on the belt are of importance. These effects include the normal (radial) and friction forces. The pulley is modeled as an axially rigid framework with zero moment of inertia so that its elements apply only normal forces to the belt. The frictional force on the belt can be computed from the relationship $T_e = T_2 e^{\mu\theta}$,[1] (where T_e is the applied total force, T_2 is the slack-side total force on the belt, μ is the coefficient of friction, θ is the total angle of contact of belt and pulley, and e the Naperian logarithm) and then used in the analysis as an external force.

Fifth, if a straight part of the belt is considered to be bent around the pulley the resulting large displacements lead to geometrical nonlinearity. To simulate this condition with a linear analysis, a circular shape is assumed for the belt and a prestrain applied to it by means of node temperatures so that the strains and stresses are the same as those in the actual bending.

For an arc of the bent section with the angle of β, the neutral line is assumed to be at the midpoint of the carcass, with length L_n and radius R_n (note that it is assumed that the pretension force in the belt causes the neutral line to remain at the midpoint of the section). The length L_n remains unchanged during bending, but points above it are in tension while those below are in compression. The amount

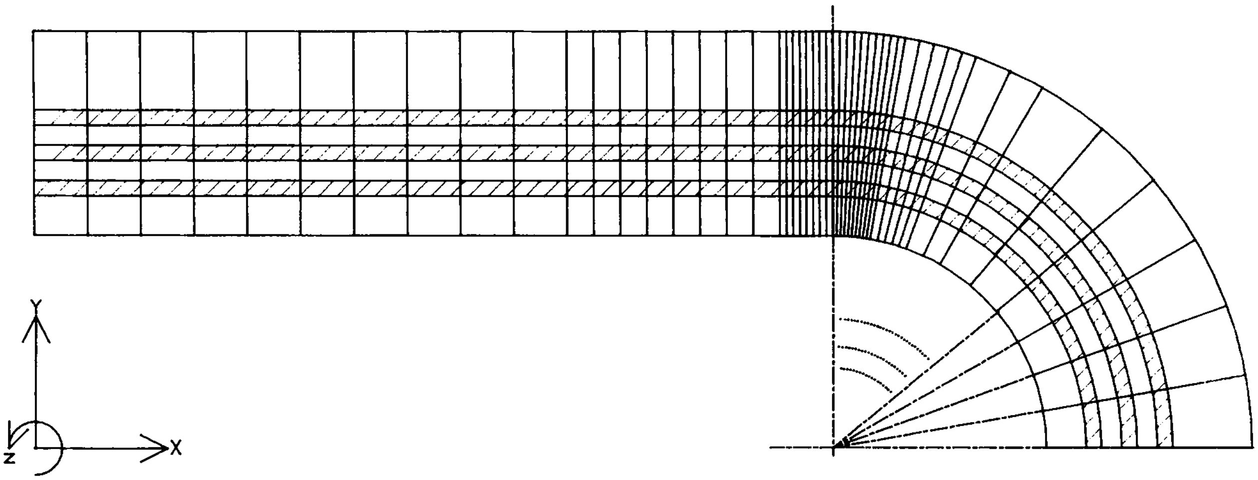

Figure 1 Schematic of the selected model (not to scale).

of strain for any length L and radius R is calculated as

$$L_n = R_n * \beta \quad (3)$$

and

$$L = R * \beta \quad (4)$$

so

$$L/L_n = R/R_n \rightarrow (L - L_n)/L_n = (R - R_n)/R_n = \epsilon \quad (5)$$

Now, if a temperature gradient of $(R - R_n)$ and a thermal expansion coefficient equal to $1/R_n$ is envisaged then the amount of required prestrain to be radially inserted can be found from eq. (5). Thus, the closer the element to the pulley surface the hotter it is considered to be.

Finally, assuming uniform conditions across the width of the belt (i.e., neglecting the boundary conditions at the sides) a two-dimensional analysis can safely be used.

Such deductions are worthy of consideration and would probably benefit from further research.

MODELING

A model of a three-ply belt consisting of three layers of fabric, two layers of rubber interplies, and top and bottom rubber covers is shown in Figure 1 together with the coordinate system. From preliminary analytical studies, it was found that the highest shear stress was at the first point of contact of the belt and the pulley.[1] The mesh at this point is therefore dense and becomes coarser as the point travels further along the straight portion of the belt.

Since the frictional force and the force difference at the pulley entrance and exit have no significant effects on the shear stresses and force differences in the layers, the frictional forces were omitted from the model. Thus, the belt becomes symmetrical

Table I Specificications Used in Analysis

Belt type	EP315	EP400	EP500	EP630
Fabric type	EP100	EP125	EP160	EP200
Fabric gauge (mm)	0.57	0.65	0.75	0.85
Fabric modulus (kgf/mm^2)	200	180	140	130
Fabric Poisson ratio	0.0	0.0	0.0	0.0
Interply rubber gauge (mm)	0.77	0.85	0.83	0.71
Rubber modulus (kgf/mm^2)	0.69	0.69	0.69	0.69
Rubber Poisson ratio	0.485	0.485	0.485	0.485
Pulley radius (mm)	150	200	225	250
Pulley (frame) modulus (kgf/mm^2)	2.1E4	2.1E4	2.1E4	2.1E4

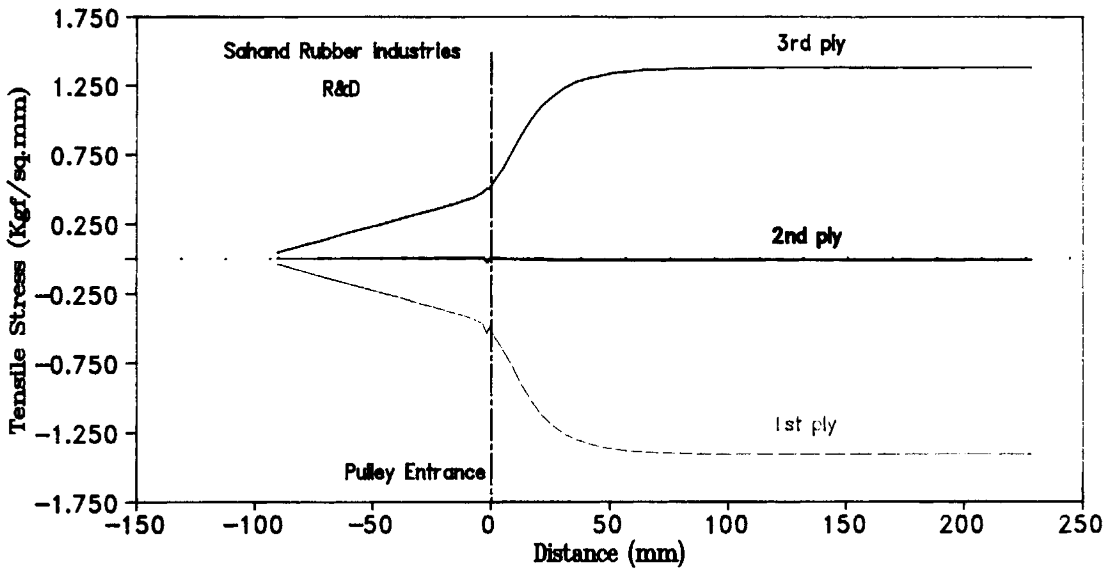

Figure 2 Stresses in three plies for Belt EP315/3 (interply th. = 0.77 mm).

about the axis passing through the midpoint of the pulley entrance and exit and the size of the model reduces to one half. This also reduces the computer run time.

All the elements of the pulley as a framework are restrained at the center in all directions. The nodal displacements at the far end of the straight section of the belt are restrained in the X direction with zero nodal rotation around the Z axis. The points at the other end of the belt have similar restraints except that nodal displacements are restrained in the Y direction. Table I indicates the characteristics of the belts analyzed.

RESULTS AND DISCUSSION

Figure 2 shows the profile of tensile stresses in three plies of EP315/3. The shear stress profiles of the rubber interplies in the same belt are indicated in Figure 3. The curve of first interply is coincident

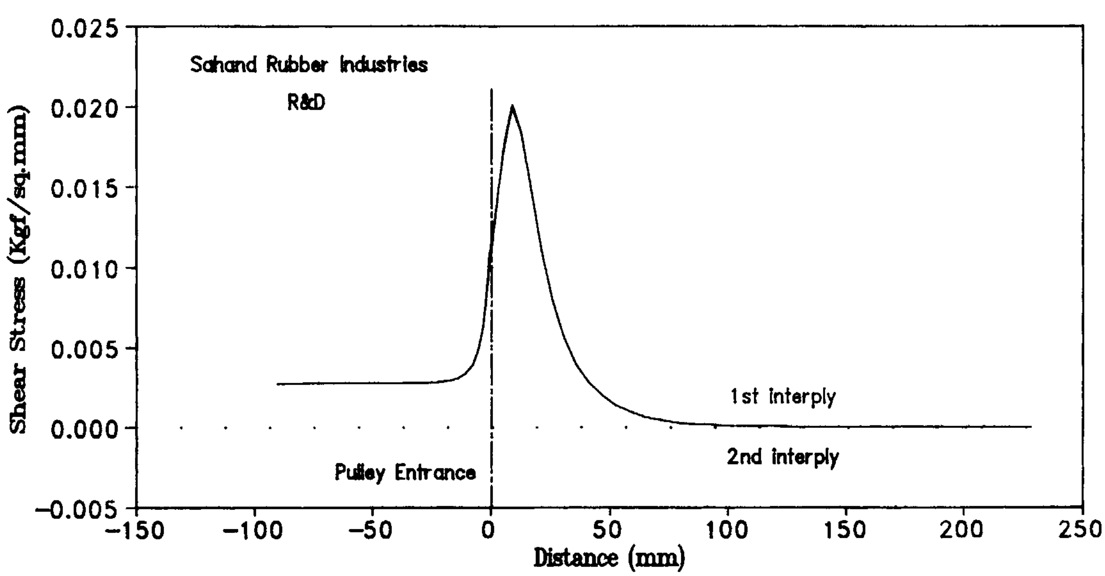

Figure 3 Shear stress in two interplies for Belt EP315/3 (interply th. = 0.77).

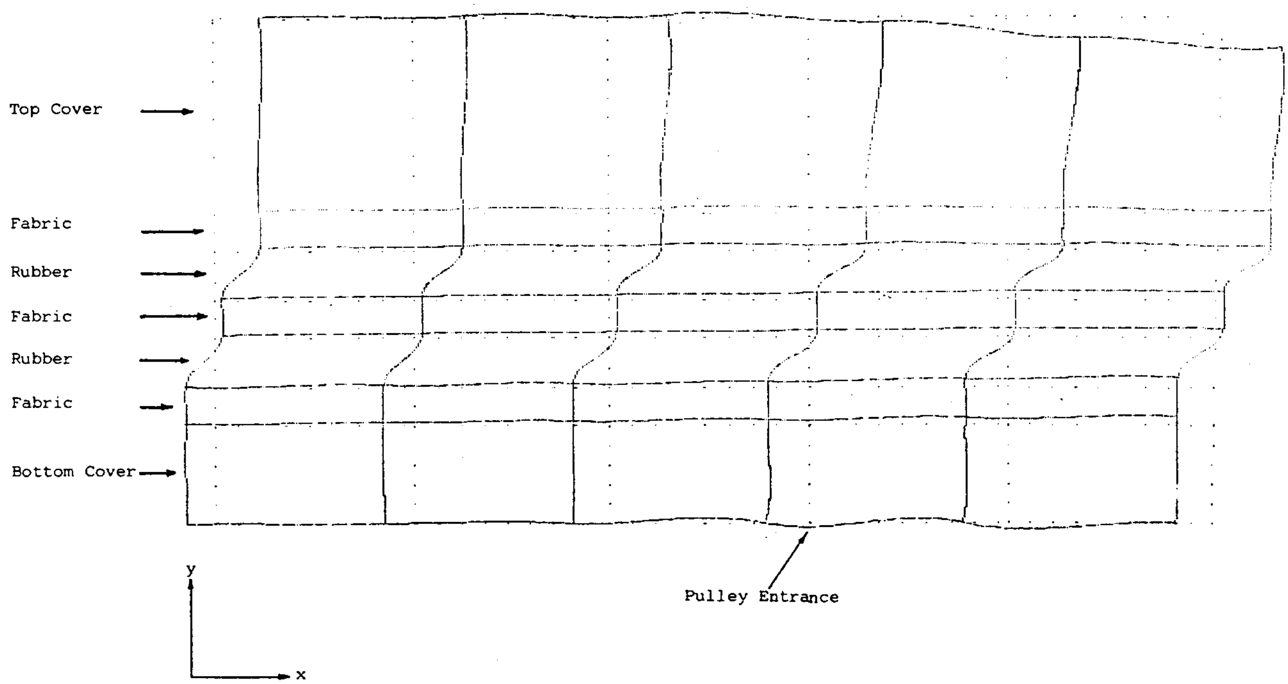

Figure 4 Exaggerated deformation of the belt at pulley entrance.

with the second. As the number of elements increases, the curves become smoother but the positions of the maxima remain unchanged.

Figure 4 indicates the relative deformation of the elements at pulley entrance. It is obvious that the amount of shear deformation of fabric relative to that of the rubber is negligible. Such assumption in closed-form solutions is, therefore, not invalid.

Figure 5 shows the maximum shear stress in the vicinity of pulley entrance for different values of the rubber Poisson ratio. The change in values do not alter the position of the highest shear stress except that the value of 0.49 and above indicate ill conditioning of the solution. Selecting the Poisson ratio equal to 0.485 prevents this situation.

Variation of maximum shear stress with interply thickness (Fig. 6) shows a minimum for all types of belts. The thickness of the interply at minimum shear stress increases with the thickness of the fabric. It is worth mentioning that in our previous study

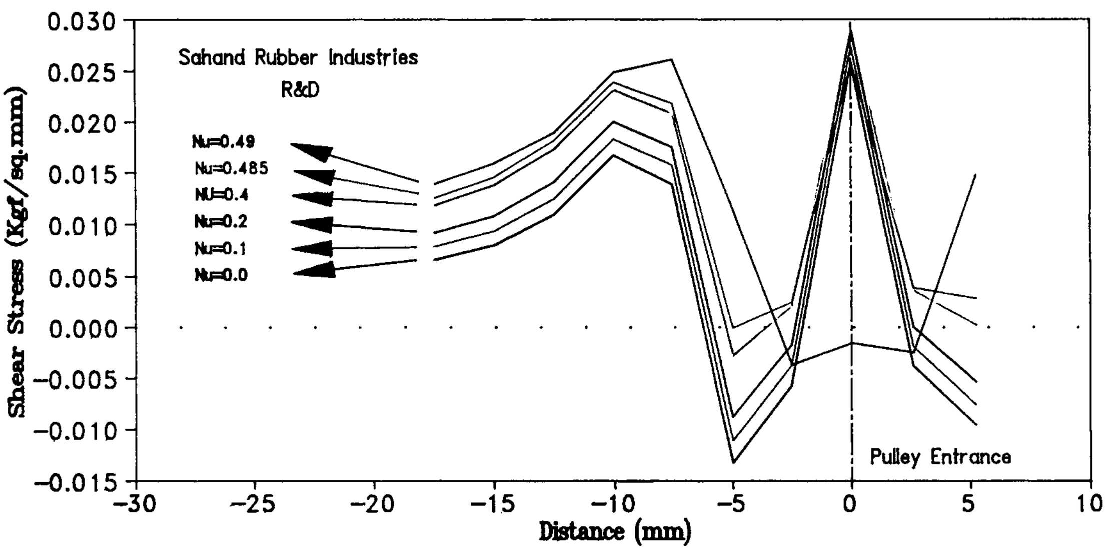

Figure 5 The variation of shear stress with Poisson's Ratio of rubber -EP315/3.

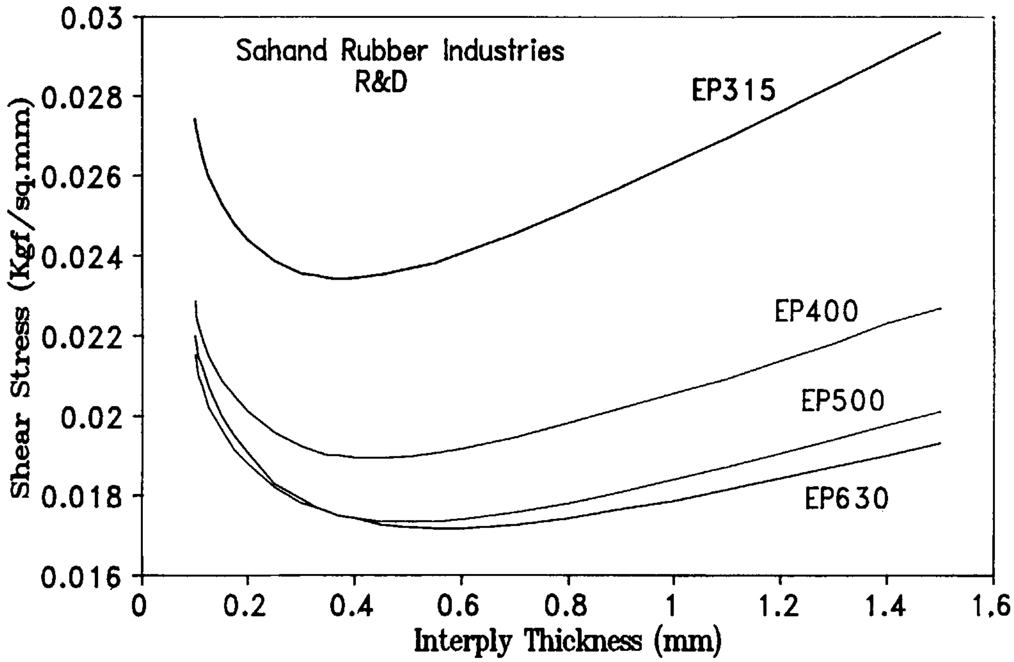

Figure 6 Shear stress vs. interply thickness for different conveyor belts by FEA.

(analytical approach) minimum shear stresses occurred at interply thickness equal to that of the fabric.[1] FEA thus predicts somehow lower values for the interply thickness.

The linear relationship of force difference between adjacent plies with interply thickness (Fig. 7) was also observed in closed-form solution.[1]

CONCLUSION

Finite element analysis can be used to evaluate the relative behavior of variable parameters in conveyor belt design. Although not totally reliable, the values obtained may be suitable for comparison purposes.

Rubber interply thickness at minimum shear

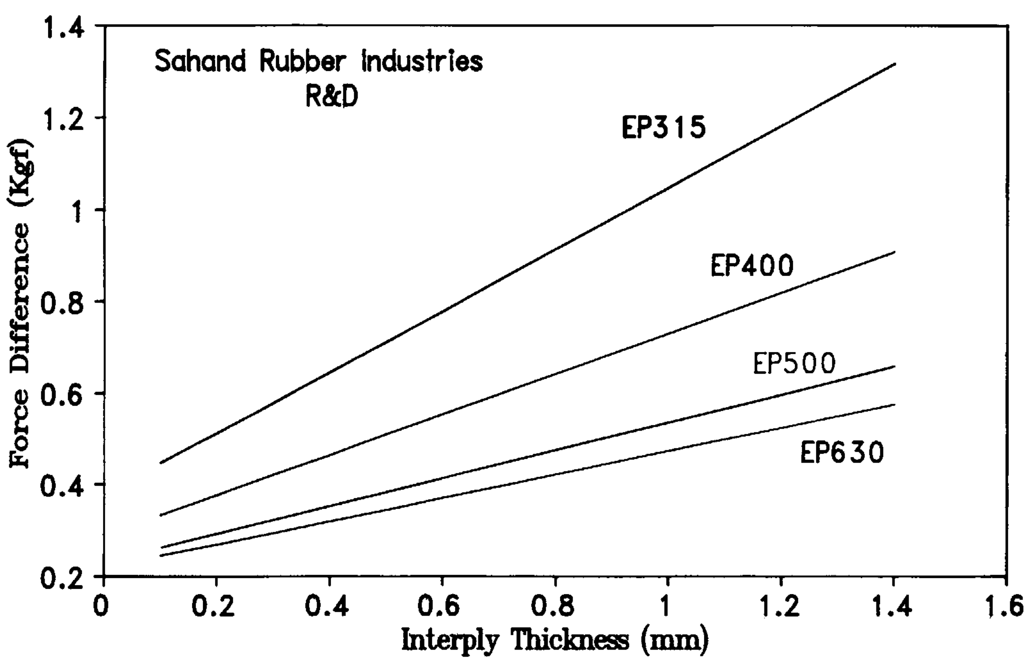

Figure 7 Force difference of adjacent plies for different conveyor belts by FEA.

stress has been shown to depend upon the thickness of the fabric used. Variation of the force difference between adjacent plies with interply thickness should also be considered in optimizing the latter. Directly related are economic considerations, as well as the process capabilities and limitations of the belt producer.

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